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Stability Analysis of an HIV/AIDS Epidemic Fractional Order Model with Screening and Time Delay

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In this paper a non linear mathematical model with fractional order $\propto$, $0 < \propto \leq 1$ is presented for analyzing and controlling the spread of HIV/AIDS. Both the disease-free equilibrium E_0 and the endemic equilibrium E^* are found and their stability is discussed using the stability theorem of fractional order differential equations. The basic reproduction number R_0 plays an essential role in the stability properties of our system. When $R_0<1$ the disease-free equilibrium E_0 is attractor, but when $R_0> 1$, E_0 is unstable and the endemic equilibrium E^* exists and it is an attractor. The effect of time delay (τ) on the screening of HIV positives that do not know they are infected is discussed. Finally numerical Simulations are also established to investigate the influence of the system parameter on the spread of the disease.

Introduction

AIDS (Acquired Immune Deficiency syndrome) is a serious life threatening disease caused by human immune deficiency virus (HIV) which is discovered in 1981 in USA. AIDS is incurable disease that has high mortality rate (kills more than 25 million worldwide per year) also it spread quickly affecting about 14,000 new case/day. Developing of AIDS takes about 6 month to 15 year. The virus attack and destruct CD_4^+ T-cell ending of loss of cell mediated immunity. The virus is transmitted through unprotected sexual contact, blood product route by sharing contaminated needle or transfusion of infected blood, also it can be transmitted from mother to her child during pregnancy, lactation, or during birth. Death generally within two years or less due to opportunistic infections often due to wide spread malignancy. Applied mathematicians have a great interest to study the HIV/AIDS dynamics spread to help biologists to find the appropriate treatment for infected humans, they seek to eliminate this threat to humanity. Mathematical models are important tools in analyzing the spread and control of HIV/AIDS as they provide short and long term prediction of HIV and AIDS incidences.

Many models available in the literature represent dynamics of disease by system of non linear ordinary differential equations without time delay [1-11]. However, inclusion of delays in fractional differential equations models makes them more realistic. In particular, Ram Naresh et al. [5] have proposed and analyzed a nonlinear mathematical model to study the effect of time delay in the recruitment of infected persons on the transmission dynamics of HIV/AIDS .Tripathi et .al. [6] have proposed a nonlinear model to study the effect of screening of unaware infectives on the spread of HIV/AIDS in a homogenous population with constant immigration of susceptibles. Sarah Al-sheikh et al. [7] have studied the local and global stability for the non linear system of ordinary differential equations of HIV/AIDS. They also studied the effect of screening of unaware infectives on the spread of HIV disease. Srinivasa Rao [8] presented a theoretical framework for transmission of HIV/AIDS epidemic in India. Cai et al. [11] investigated an HIV model with treatment, they established the model with two infective stages and proved that the dynamics of the spread of the disease are completely determined by the basic reproduction number R_0.

We will study the nonlinear model with a fractional order $\propto, 0 <\propto\leq 1$. In recent years fractional calculus has an interest to mathematicians as it has many engineering and medical applications. There is more than one definition for the fractional derivative [12,13]. In 1867, Grünewald-Letnicov defined the fractional derivative as,

$$D^{\propto}f(x) = \lim_{N\to\infty}\left(\frac{1}{h^{\propto}}\right)\sum_{k=0}^{N}\frac{\Gamma(k-\propto)}{\Gamma(-\propto)\Gamma(k+1)}.f(x-kh)\ ,n-1<\propto\leq n$$

In 1967 caputo defined the fractional derivative of a function f(x) as,

$$D^{\propto}f(x) = \frac{1}{\Gamma(n-\propto)}\int_a^x (x-t)^{n-\propto-1}\frac{d^n}{dx^n}f(t)\ dt \quad ,n-1<\propto< n$$

In this paper we will generalize AIDS/HIV model to a fractional order system of order $\propto$ in sense of caputo definition because it is equivalent to ordinary differential equation when $\propto= 1$. The population is divided into four sub-classes, the susceptible S(t), the infectives that do not now they are infected $I_1(t)$, the infectives that know they are infected $I_2(t)$ (by means of medical screening or otherwise) and the AIDS population A(t), then the model will be,

$$\begin{aligned}
D^{\alpha}S(t) &= Q_0-(\beta_1 I_1(t)+\beta_2 I_2(t))S(t)-\mu S(t)\\
D^{\alpha}I_1(t) &= (\beta_1 I_1(t)+\beta_2 I_2(t))S(t)-(\theta+\delta+\mu)I_1(t)\\
D^{\alpha}I_2(t) &= \theta I_1(t)-(\delta+\mu)I_2(t)\\
D^{\alpha}A(t) &= (I_1(t)+I_2(t))\delta-(d+\mu)A(t)
\end{aligned} \tag{1}$$

Where all the parameters have a constant values defined as: Q_0 is the rate of immigration of susceptible, β_1 is the per capita rate for susceptibles individuals with unaware infectives, β_2 is the per capita rate for susceptibles individuals with aware infectives, μ is the natural mortality rate unrelated to AIDS, θ is the rate of unaware Infectives to become aware infectives by screening, δ is the rate by which types of infectives develop AIDS and d is the AIDS related death rate. It is clear that the variable A(t) does not appear in the first three equations, so reducing system (1) we get:

$$\begin{aligned}
D^{\alpha}S(t) &= Q_0-(\beta_1 I_1(t)+\beta_2 I_2(t))S(t)-\mu S(t)\\
D^{\alpha}I_1(t) &= (\beta_1 I_1(t)+\beta_2 I_2(t))S(t)-(\theta+\delta+\mu)I_1(t)\\
D^{\alpha}I_2(t) &= \theta I_1(t)-(\delta+\mu)I_2(t)
\end{aligned} \tag{2}$$

Clearly if $\propto= 1$, the system will be the nonlinear ordinary differential equations as presented in [5-7]. The region of stability of the fractional order system as reviewed in [12-14] is the region in which the system eigenvalues λ of the characteristic equation obtained from the Jacobian matrix of system (2) at a certain equilibrium point satisfies that, $|\arg(\lambda)| > \alpha\pi/2$.

In this work we discuss the stability of the HIV/AIDS model according to the relations between the system parameters. We also study the behavior of our system when there is a time delay on the screening.

This paper is organized as the following: the Equilibria of the model and their stability properties are presented in section 2. In section 3 the effect of existence a time delay on the screening on the stability behavior of the model is discussed. Real life examples with their numerical solutions are given in section 4, and finally we gave our conclusion in section 5.

Equilibria and Their Stability

The Equilibrium Points of system (2) are obtained by solving the nonlinear algebraic equations

$$D^{\propto}S(t) = D^{\propto}I_1(t) = D^{\propto}I_2(t) = 0 \tag{3}$$

System (2) has free equilibrium point $E_0(\frac{Q_0}{\mu},0,0)$ if $R_0< 1$, while if $R_0> 1$ there is in addition to E_0, a positive endemic equilibrium $E^*(S^*,I_1^*,I_2^*)$ where R_0 is the basic reproduction number defined in [9,5] as:

$$R_0 = \frac{Q_0[\beta_1(\delta+\mu)+\beta_2\theta]}{\mu(\delta+\mu)(\theta+\delta+\mu)}, \tag{4}$$

and $I_1^* = \frac{Q_0(R_0-1)}{R_0(\theta+\delta+\mu)}$, $I_2^* = \frac{\theta}{(\delta+\mu)} I_1^*$, and $S^* = \frac{(\delta+\mu)(\theta+\delta+\mu)}{\beta_1(\delta+\mu)+\beta_2\theta}$.

The following theorem defines the stability behavior of system (2) around the free Equilibrium point E_0.

Theorem 1 System (2) will be locally asymptotically stable around E_0 if $R_0 < 1$, and unstable if $R_0 > 1$.

Proof Since the characteristic equation of the Jacobian matrix for system (2) around E_0 is:

$$(-\mu - \lambda)(\lambda^2 + q_1\lambda + q_2) = 0 \tag{5}$$

Where $q_1 = \theta + 2\delta + 2\mu - \beta_1 Q_0/\mu$, $q_2 = (\theta + \delta + \mu)(\delta + \mu)(1 - R_0)$.

The eigenvalues of Eq.(5) are $\lambda_1 = -\mu$, and the roots of the quadratic Equation

$$\lambda^2 + q_1\lambda + q_2 = 0 \tag{6}$$

If $R_0 < 1$, then $q_2 > 0$ and also $\mu(\delta + \mu)(\theta + \delta + \mu) > Q_0\beta_1(\delta + \mu) + Q_0\beta_2\theta > Q_0\beta_1(\delta + \mu)$ which means that $(\theta + \delta + \mu) > \frac{\beta_1 Q_0}{\mu}$, then $q_1 > 0$. Then applying Routh-Hurwitz criteria, insure that E_0 is locally asymptotically stable. If $R_0 > 1$ then $q_2 < 0$, and there is one positive Real root for Eq. (6), thus E_0 will be unstable.

Lemma1 System (2) will be locally stable around E_0 if $R_0 = 1$.

Proof Since $R_0 = 1$, then $q_2 = 0$, $q_1 > 0$ then the roots of Equation (6) will be $\lambda_2 = 0$, $\lambda_3 = -q_1$ so the system will be locally stable.

Now we will discuss the stability of the positive equilibrium E^*.

Definition 1 [14] The discriminate $D(P)$ of a polynomial $P(\lambda) = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$ is defined by

$$D(P) = 18a_1a_2a_3 + (a_1a_2)^2 - 4a_3(a_1)^2 - 4(a_2)^3 - 27(a_3)^2. \tag{7}$$

Let $k = (\theta + \delta + \mu) = \beta_1 S^* + \beta_2 I_2^* S^*/I_1^*$. The characteristic Equation of system (2) around E^* is:

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{8}$$

Where

$$a_1 = \delta + 2\mu + kI_1^*/S^* + k - \beta_1 S^*$$

$$a_2 = (k - \beta_1 S^* + \delta + \mu)\left(\mu + \frac{kI_1^*}{S^*}\right) + k\beta_1 I_1^* \tag{9}$$

$$a_3 = kI_1^*(\beta_1(\delta + \mu) + \beta_2\theta) > 0.$$

Theorem 2 Consider $R_0 > 1$ in system (2), then the epidemic point E^* will be asymptotically stable if :

$$D(P) > 0 \ , a_1a_2 > a_3, \propto \in (0,1] \tag{10}$$

$$\text{Or } D(P) < 0 \ , \text{and } \propto \in [0, \tfrac{2}{3}) \tag{11}$$

Where $D(P)$, a_1, a_2 and a_3 are defined in (7, 9).

Proof For $D(P) > 0$, $a_1a_2 > a_3$, and $k > \beta_1 S^*$, then $a_1 > 0$, $a_3 > 0$, using Routh-Hurwitz criteria, then $|\arg(\lambda)| > \alpha\pi/2$ and the system will be locally asymptotically stable around E^*. When $D(P) < 0$, $\propto \in [0, \frac{2}{3})$, since the value$(k - \beta_1 S^*)(\delta + \mu) - \beta_2\theta S^* = 0$, we get $a_2 > 0$, then the conditions for stability of the fractional order system are satisfied [15], then E^* is locally asymptotically stable.

System with Delay Time on the Screening $(\tau > 0)$

Consider the system:

$$
\begin{aligned}
D^{\alpha} S(t) &= Q_0 - (\beta_1 I_1(t) + \beta_2 I_2(t))S(t) - \mu S(t) \\
D^{\alpha} I_1(t) &= (\beta_1 I_1(t) + \beta_2 I_2(t))S(t) - (\delta+\mu) I_1(t) - \theta I_1(t-\tau) \\
D^{\alpha} I_2(t) &= \theta I_1(t-\tau) - (\delta+\mu) I_2(t)
\end{aligned} \tag{12}
$$

Where τ is the delay time on the screening. To discuss the stability of the free disease equilibrium point E_0, we examine the characteristic Equation of the Jacobian matrix of system (12) that given by,

$$(-\mu-\lambda)\left[\lambda^2 + u_1\lambda + u_2 + e^{-\lambda\tau}(b_1\lambda + b_2)\right] = 0 \tag{13}$$

Where $u_1 = 2(\delta+\mu) - \beta_1 Q_0/\mu$, $u_2 = (\delta+\mu)[(\delta+\mu) - \beta_1 Q_0/\mu]$, $b_1 = \theta$,

$b_2 = \theta\left[\delta + \mu - \frac{Q_0\beta_2}{\mu}\right]$. The eigenvalues are $\lambda_1 = -\mu$ and the solutions of the equation

$$\lambda^2 + u_1\lambda + u_2 + e^{-\lambda\tau}(b_1\lambda + b_2) = 0 \tag{14}$$

First we will approximate a value for the critical time (τ_c) over which the system will be unstable. Let $e^{-\lambda\tau} = 1 - \lambda\tau$, substituting in (14) we get:

$$\lambda^2 + B_1\lambda + B_2 = 0 \tag{15}$$

where $B_1 = (u_1 + b_1 - b_2\tau)/(1 - b_1\tau)$, $B_2 = (u_2 + b_2)/(1 - b_1\tau)$.

At $\lambda = re^{i\frac{\propto\pi}{2}}$, then τ will be the critical time τ_c. Substituting in (15) and separating the real and imaginary parts we have,

$$r^2\cos(\propto\pi) + B_1 r\cos(\propto\pi/2) + B_2 = 0 \tag{16}$$

$$B_1 r\sin(\propto\pi/2) + r^2\sin(\propto\pi) = 0 \tag{17}$$

Multiplying (16) by $\sin(\propto\pi)$, (17) by $\cos(\propto\pi)$ and subtracting, then

$$r = -2\left(\frac{B_2}{B_1}\right)\cos(\propto\pi/2) \tag{18}$$

The positive values for r at $\frac{B_2}{B_1} < 0$ which gives

$$\tau < (b_1 + u_1)/b_2 \text{ if } b_2 < 0, \text{ and } \tau > (b_1 + u_1)/b_2 \text{ if } b_2 > 0.$$

Substituting by the values of r, B_1, B_2 in (17) we get,

$$\tau_c = \left(-c_1 + \sqrt{{c_1}^2 - 4c_2}\right)/2, {c_1}^2 > 4c_2 \tag{19}$$

Where $c_1 = [4b_1 k(\delta+\mu)(1-R_0)\cos^2(\propto\frac{\pi}{2}) - 2b_2(b_1+u_1)]/{b_2}^2$, $c_2 = [(b_1+u_1)^2 - 4k(\delta+\mu)(1-R_0)\cos^2(\propto\frac{\pi}{2})]/{b_2}^2$

Corollary 1 System (12) will be asymptotically stable around the free Equilibrium point E_0 if $R_0 < 1$ and the delay time $\tau < \tau_c$. Where τ_c is defined by (19).

Stability Behavior of the Point E^*

Since the characteristic Equation of the Jacobian matrix of system (12) around E^* is:

$$\lambda^3 + a_{11}\lambda^2 + a_{22}\lambda + a_{33} + e^{-\lambda\tau}[b_{11}\lambda^2 + b_{22}\lambda + b_{33}] = 0 \tag{20}$$

where

$a_{11} = \mu R_0 + 2(\delta+\mu) - \beta_1 Q_0/\mu R_0$, $a_{22} = (\delta+\mu)[2\mu R_0 + \delta + \mu] - \frac{\beta_1 Q_0}{R_0}(2 + \frac{\delta}{\mu})$, $a_{33} = (\delta+\mu)\left[\mu R_0(\delta+\mu) - \frac{\beta_1 Q_0}{R_0}\right]$

$$b_{11} = \theta,\ b_{22} = \theta\left[\mu R_0 + (\delta+\mu) - \frac{\beta_2 Q_0}{\mu R_0}\right],\ b_{33} = \theta\left[\mu R_0(\delta+\mu) - \frac{\beta_2 Q_0}{R_0}\right].$$

Now we try to get an approximate formula for the critical delay time (τ_c) over which the system will be unstable. Similarly put $e^{-\lambda\tau} = 1 - \lambda\tau$ in (20) then,

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0 \tag{21}$$

where $A_1 = \frac{a_{11}+b_{11}-b_{22}\tau}{1-b_{11}\tau}$, $A_2 = \frac{a_{22}+b_{22}-b_{33}\tau}{1-b_{11}\tau}$, $A_3 = \frac{a_{33}+b_{33}}{1-b_{11}\tau}$

We analyzed the stability conditions for the characteristic Equation (21) to find a relation for the time delay τ to conclude the following lemma,

Lemma 2 Consider system (12) is asymptotically stable around E^* at $D(p) > 0, Q_1^2 > 4Q_2$, and

$$a_{33} + b_{33} < \min\left\{(a_{11} + b_{11})(a_{22} + b_{22}), \frac{b_{22}(a_{22}+b_{22})+b_{33}(a_{11}+b_{11})}{b_{11}}\right\} \tag{22}$$

then the time delay τ satisfies one of the following conditions:

$$0 < \tau < \min\left\{\frac{1}{b_{11}}, \frac{a_{11}+b_{11}}{b_{22}}, \tau_1, \tau_2\right\}, \text{ if } b_{22}, b_{33} > 0, \tag{23}$$

$$0 < \tau < \min\left\{\frac{1}{b_{11}}, \tau_1, \tau_2\right\}, \text{ if } b_{22}, b_{33} < 0, \tag{24}$$

$$\max\{0, \tau_2\} < \tau < \min\left\{\frac{1}{b_{11}}, \tau_1\right\}, \text{ if } b_{22} < 0, b_{33} > 0, \tag{25}$$

$$\text{And, } \max\{0, \tau_2\} < \tau < \min\left\{\frac{1}{b_{11}}, \tau_1, \frac{a_{11}+b_{11}}{b_{22}}\right\}, \text{ if } b_{22} > 0, b_{33} < 0 \tag{26}$$

$$\text{Where } \tau_{1,2} = \frac{1}{2}\left[-Q_1 \pm \sqrt{Q_1^2 - 4Q_2}\right], Q_1 = \frac{b_{33}[a_{33}+b_{33}-a_{11}-b_{11}]-b_{22}(a_{22}+b_{22})}{a_{22}.b_{33}}, Q_2 = \frac{(a_{11}+b_{11})(a_{22}+b_{22})-(a_{33}+b_{33})}{a_{22}.b_{33}}. \tag{27}$$

Proof Since system (12) is asymptotically stable when $D(p) > 0$, then Routh-Hurwitz are satisfied for equation (21),

So $A_1 > 0$ which gives, $\tau < \frac{1}{b_{11}}$ *and* $\tau < \frac{a_{11}+b_{11}}{b_{22}}$ at $b_{22} > 0$ or $\tau > 0$ at $b_{22} < 0$ since $a_{11} + b_{11} > 0$,

$A_3 > 0$ gives, $\tau < \frac{1}{b_{11}}$ since $a_{33} + b_{33} > 0$, and $A_1A_2 > A_3$ which gives, $(a_{11} + b_{11} - b_{22}\tau)(a_{22} + b_{22} - b_{33}\tau) - (a_{33} + b_{33})(1 - b_{11}\tau) > 0$, At $b_{22}b_{33} > 0$ then $\tau^2 + Q_1\tau + Q_2 > 0$ from which $\tau < \tau_{1,2}$, and if $b_{22}b_{33} > 0$ then $\tau^2 + Q_1\tau + Q_2 < 0$ which gives, $\tau_2 < \tau < \tau_1$.

Then τ satisfies the values (23)-(26) at the different values of b_{22}, b_{33}. It is clear that $\tau_{1,2} \in R^+$ at $Q_1^2 > 4Q_2$,

$Q_1 < 0$, and $Q_2 > 0$, Which satisfies (22). where $Q_1, Q_2, \tau_{1,2}$ are defined in (27).

Lemma 3. Assume that system (12) is asymptotically stable around E^* at

$D(p) < 0, \propto \in [0, \frac{2}{3})$, then the time delay τ satisfies one of the following conditions:

$$0 < \tau < \min\left\{\frac{1}{b_{11}}, \frac{a_{11}+b_{11}}{b_{22}}, \frac{a_{22}+b_{22}}{b_{33}}\right\}, \text{ at } b_{22} > 0, b_{33} > 0, \tag{28}$$

$$\max\{0, \frac{a_{11}+b_{11}}{b_{22}}, \frac{a_{22}+b_{22}}{b_{33}}\} < \tau < \frac{1}{b_{11}}, \text{ at } b_{22} < 0, b_{33} < 0, \tag{29}$$

$$\max\{0, \frac{a_{22}+b_{22}}{b_{33}}\} < \tau < \min\left\{\frac{1}{b_{11}}, \frac{a_{11}+b_{11}}{b_{22}}\right\}, \text{ at } b_{22} > 0, b_{33} < 0, \tag{30}$$

$$\text{and } 0 < \tau < \min\left\{\frac{1}{b_{11}}, \frac{a_{22}+b_{22}}{b_{33}}\right\}, \text{ at } b_{22} < 0, b_{33} > 0 \tag{31}$$

Proof Applying the stability conditions in [14] for (21), at $D(p) < 0, \propto \in [0, \frac{2}{3})$ then,

$A_1 \geq 0$, which gives $\tau \leq \frac{a_{11}+b_{11}}{b_{22}}$ at $b_{22} > 0$, *and* $\tau \geq \frac{a_{11}+b_{11}}{b_{22}}$ at $b_{22} < 0$, $A_2 \geq 0$, then $\tau \leq \frac{a_{22}+b_{22}}{b_{33}}$ at $b_{33} > 0$, *and*

$\tau \geq \frac{a_{22}+b_{22}}{b_{33}}$ at $b_{33} < 0$, and $A_3 > 0$ gives, $\tau < \frac{1}{b_{33}}$. Then τ satisfies (28)-(31) at the different values of b_{22}, b_{33}. So we can conclude the following theorem to ensure the stability of system (12) around E^*.

Theorem 3 System (12) will be asymptotically stable around the endemic point E^* at $R_0 > 1$, if (10) or (11) are satisfied and the delay time τ satisfies the inequalities (23)-(26) or (28)-(31) respectively.

Numerical Simulations

In this section we give some illustrative examples to verify the obtained results on systems (2) and (12). We use the non-standard finite difference method [13,15] that use the Grünewald-Letnicov descretization method for solving the following examples using Matlab program.

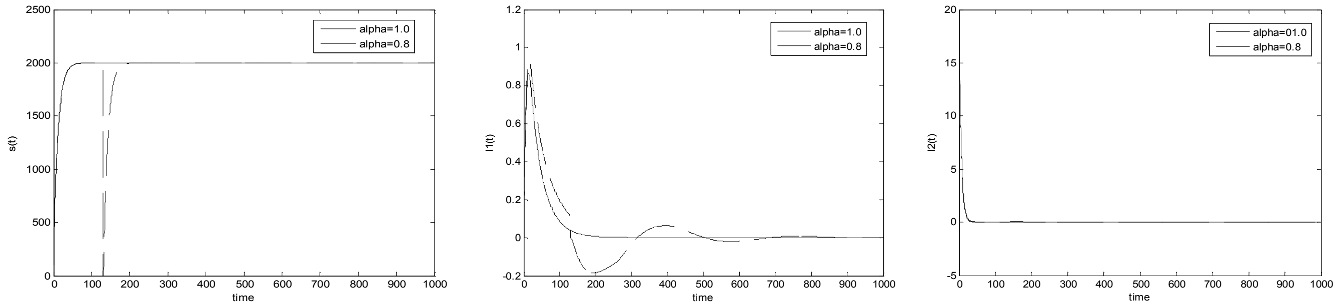
Example 1: Consider the parameters of system (2) have the following values: $Qo = 200, \mu = 0.1,\ \delta = 0.1,\ \theta = 0.015,$

$\beta_1 = 0.00009,\ \beta_2 = 0.000027$, Figure 1 (a, b, c) gives the numerical solution of $S(t), I_1(t), I_2(t)$. Hence $R_0 = 0.8560$ and the unique equilibrium point $E_0 = (2000,0,0)$ is asymptotically stable, this results enhance theorem 1.

Example 2: Let the parameters of system (12) are: $Qo = 1200,\ \mu = 0.019,\ \delta = 0.52,\ \theta = 0.02,\ \beta_1 = 0.00009,\ \beta_2 = 0.000027$, then $R_0 = 10.2817$. Figure 2 (a, b, c) gives the time response of $S(t), I_1(t)$ and $I_2(t)$ when $\tau = 0, \tau = 40$ years and $\propto = 0.5$, this results are the same that obtained by theorem 2, 3.

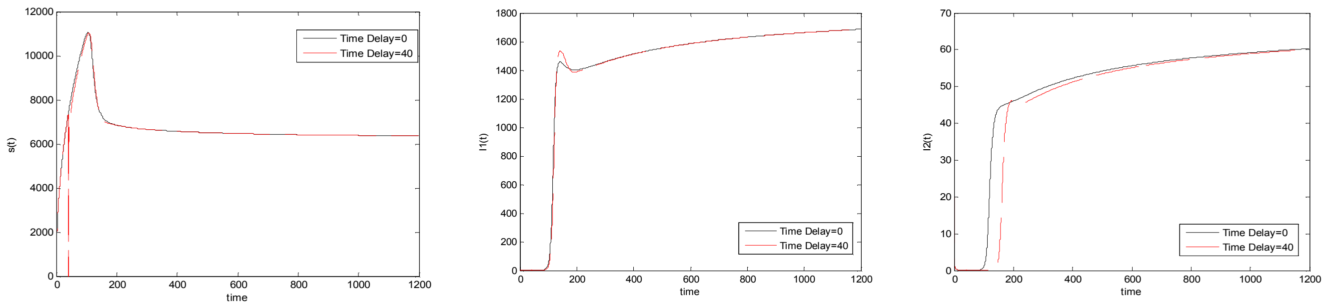
Example 3: assume that system (12) has the parameters: $Qo = 500, \mu = 0.01, \delta = 0.3, \theta = 0.025, \beta_1 = 0.00009, \beta_2 = 0.000027$, at the time delay $\tau = 0, \tau = 35$ and $\propto = 0.9$, Figure 3 (a, b, c) shows that the solution curves tends to the positive equilibrium E^* where R_0=13.7578, theorem 2,3 garanteethe obtained results.

Example 4: Finally using the parameters: $Qo = 800, \mu = 0.01, \delta = 0.5, \theta = 0.03, \beta_1 = 0.00009, \beta_2 = 0.000027$ at the time delay $\tau = 0, \tau = 30$, and $\propto = 0.9$, then $R_0 = 13.5686$. Figure 4 (a, b, c) represents the time response of $S(t), I_1(t), I_2(t)$.



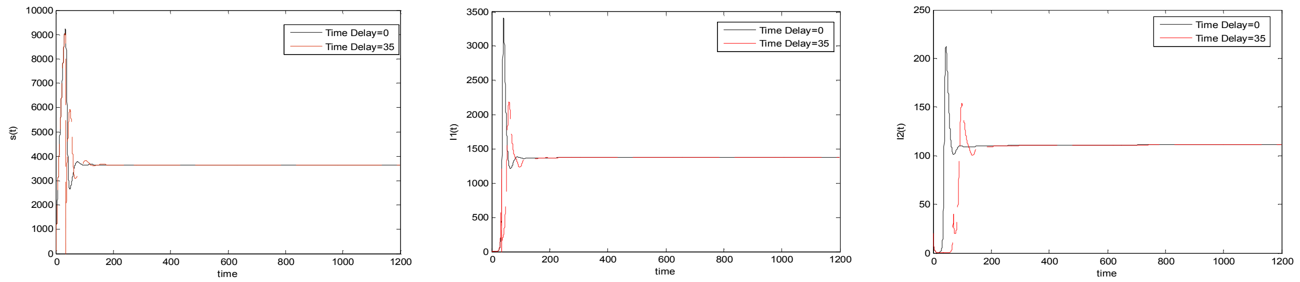
(a) the time response of S(t) (b) the time response of $I_1(t)$ (c) the time response of $I_2(t)$

Figure 1. *Variation of S(t) , $I_1(t)$, $I_2(t)$ against the time,for Example 1 .*



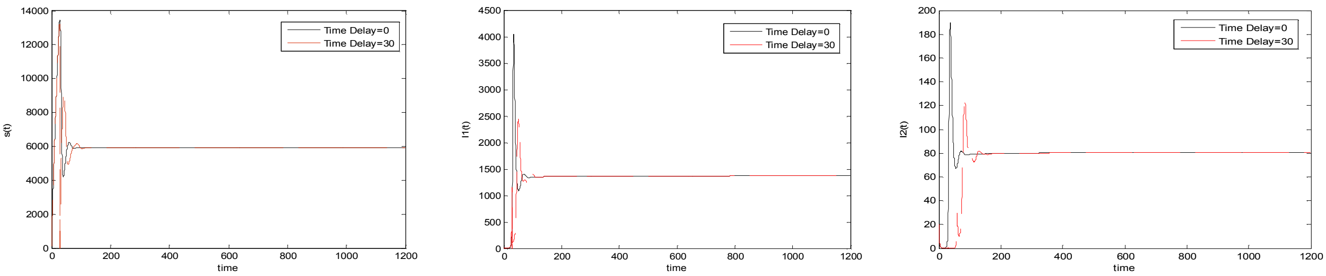
(a) the time response of S(t) (b) the time response of $I_1(t)$ (c) the time response of $I_2(t)$

Figure 2. *The time response of S(t), $I_1(t)$ and $I_2(t)$ of Example 2.*



(a) the time response of S(t) (b) the time response of $I_1(t)$ (c) the time response of $I_2(t)$

Figure 3. *Variation of S(t), $I_1(t)$, $I_2(t)$ against the time, for Example 3.*



(a) the time response of S(t) (b) the time response of $I_1(t)$ (c) the time response of $I_2(t)$

Figure 4. *Variation of S(t), $I_1(t)$, $I_2(t)$ against the time, for Example 4.*

Conclusion

In this paper, a nonlinear mathematical HIV model with fractional order α is formulated. The stability of both free and endemic equilibrium point are discussed. Sufficient conditions for local stability of the disease free equilibrium point E_0 are given in terms of the basic reproduction number R_0 of the model, where it is asymptotically stable if $R_0<1$ and the delay time τ is less than the critical delay time τ_c defined by (19). The positive infected equilibrium E^* exist when $R_0>1$ and sufficient conditions that guarantee the asymptotic stability of this point are given when $\tau \geq 0$. At $\propto = 1$, the stability behavior of system (2) will be similar to the nonlinear system of ordinary differential equations discussed in [5, 9] with similar results. Finally numerical simulations for different sets of parameters are solved and it is agree with our obtained results concerning with system (2) and (12) ■

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Zahra received his PhD in Numerical Analysis from Tanta University in 2008. Zahra is an Assist Prof of Engineering Mathematics in Tanta University. He published a lot of papers in the top international scientific journals and an active reviewer in several international scientific journals. Zahra's research has focused on numerical methods, analysis and computation, numerical solution of ordinary and partial differential equations (integer and fractional order), applications of numerical methods to science and engineering such as heat transfer problem, chemical reactor theory, aerodynamics, reaction-diffusion process, quantum mechanics, optimal control, inverse problems, etc.

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References

[1] Fatma bozkurt and Fatma Peker, Mathematical modeling of HIV epidemic and stability analysis, Advances in Difference Equations, A Springer Open Journal, doi: 10.1186/1687-1847-(2014)-95.

[2] Nita H. Shah, Jyoti Gupta, HIV/AIDS – Malaria co-infection dynamics, sciknow Research Journal of Modeling and Simulation, RJMS 2014, 1 (1), 8-13, 2014.

[3] M. O. Okongo, J. Kirimi, A. L. Murwayi and D. K. Murithi, Mathematical Analysis of a comprehensive HIV/AIDS Model: Treatment versus vaccination, Applied Math. Sci. Vol. 7, (2013), No. 54, 2687-2707.

[4] Basavarajaiah. D. M. B. Narasimhamurthy, K. Maheshappa. B. Leelavathy, Mathematical Model Approach to HIV/AIDS Transmission From Mother To Child, Int. Journal sci., Vol. 1, Issue 9, October (2012).

[5] Ram Naresh, Agraj Tripathi, and Dileep Sharma, A non linear AIDS epidemic model with screening and time delay, App. Math. Comp.217(2011), 4416-4426.

[6] Tripathi, A., Naresh, R., and Sharma, D., Modeling the effect of screening of unaware infectives on the spread of HIV infection, Appl. Math. 184(2007), 1053-1068.

[7] Sarah Al-Sheikh, Farida Musali, Muna Alsolami, stability Analysis o an HIV/AIDS Epidemic Model with screening, International Math. Forum, vol.6 (2011), no.66, 3251-3273.

[8] A.S.R. Srinivasa Rao, Mathematical modeling of AIDS epidemic in India, Curr. Sci. 84(2003), 1192-1197.

[9] Y.H. Hsieh, C.H. Chen, Modeling the social dynamics of a sex industry: its implications for spread of HIV/AIDS, Bull. Math. Biol. 66(2004), 143-249.

[10] E. Shim, A note on epidemic models with infective immigrants and vaccination, Math. Bios. Eng2 (2006), 557-566.

[11] Cai, Liming, Li, Xuezhi, Ghosh, Mini, and Guo, Baozhu, Stability analysis of an HIV/AIDS epidemic model with treatment, Journal of computational and Applied Math. 229 (2009),313-323.

[12] Kenneth S. Miller, Bertram Ross, An introduction to the fractional calculus and fractional differential equations, A wiley-Interscience Publication, USA, (1993).

[13] Igor Podlubny, fractional differential equations, Academic Press,USA, (1999).

[14] E. Ahmed, A.M.A. El-sayed, and Hala A.A. El-saka, On some Routh-Hurwitz conditions for fractional order differential equations and their applications in Lorenz, Rossler, Chua chen systems, physics letters A 358 (2006).

[15] A.G. Radwan, K. Moaddy, Shaher Momani, Stability and non-standard finite difference method of the generalized chua's circuit, Comp. and math. With Applications 62 (2011) 961-970.

[16] R. E. Mickens, nonstandard finite difference models of differential equations, World scientific, Singapore, 1994.

[17] K. Moaddy, I. Hashim, S. Momani, non-standard finite difference scheme for solving fractional-order Rossler chaotic and hyperchaotic system, Comp. and math. With Applications 62 (3) (2011) 1068-1074.